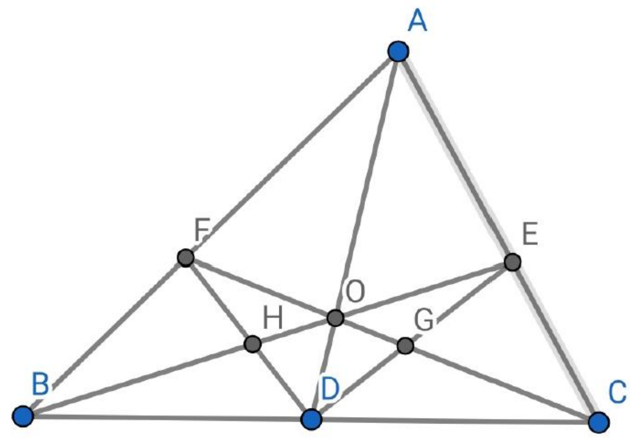
01.02.2024 – II Prize Winner – Mrs.Madhumitha's Solution



Given:

DE is angle bisector of $\angle ADC$.
BE & AD intersect at O

Construction:
Draw a line CO extend to meet AB at F
Draw DF.
Let DE intersect CF at G.
DF intersect BE at H.

Proof:
Given DG is angle bisector of $\angle ODC$

$\therefore \frac{OD}{DC} = \frac{OG}{GC}$ -------------------- (1) (Angle bisector theorem)

As AD, BE, CF are concurrent at O.
By concurrency theorem,

$\frac{CG}{GO} = \frac{CF}{FO}$ -------------------------- (2)

From (1) & (2)

$\frac{OD}{DC} = \frac{OG}{GC} = \frac{OF}{FC}$ ------------------------ (3)

DF is external angle bisector of $\angle ODC$.
i.e., DF is the bisector of ∠BDO

$\frac{OD}{DB} = \frac{OH}{HB}$ --(4)

By concurrency theorem

$\frac{BH}{HO} = \frac{BE}{EO}$ --(5)

From (5) & (6)

$\frac{OD}{DB} = \frac{OH}{HB} = \frac{OE}{BE}$ ---------------------------------------(6)

By Unit Pieces theorem,
In ΔABC, AD, BE, CF are cevians concurrent at O.

$\frac{OD}{AD} + \frac{OE}{BE} + \frac{OF}{CF} = 1$

$\frac{OD}{AD} + \frac{OD}{DB} + \frac{OD}{DC} = 1$ by (3) & (6)

ie $\frac{1}{AD} + \frac{1}{BD} + \frac{1}{CD} = \frac{1}{OD}$ ---------------------------- Hence Proved